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The Space-Time Transformation in Feynman Propagator for a Time-Dependent Harmonic Oscillator

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Abstract

We demonstrate that the use of space-time transformation in path integration can simplify the calculation of the Feynman propagator for a harmonic oscillator with time-dependent mass and frequency. We show that such a propagator can be easily obtained from the unit mass and frequency propagator in the new space-time coordinate systems.

Keywords: Path Integration, Propagator, Time-Dependent Harmonic Oscillator

Introduction

Recently there has been considerable interest in the theory of time-dependent Hamiltonians systems [1-5]. The system with the time-dependent Hamiltonians were investigated using various method. The various applications in many area of physics, such as quantum optics, cosmology, and nanotechnology are the main reasons for intensive interesting. The example of application for time-dependent harmonic oscillator are the model of expanding universe and the motion of an ion in solid state trap. S.Pepore and et.al. [6-8], applied both Feynman path integral and Schwinger method to study the propagator and wave function for a harmonic oscillator with time-dependent mass and frequency. The aim of this paper is to derive the propagator for a harmonic oscillator with time-dependent mass and frequency as described by the Hamiltonian

\[ H(t) = \frac{p^2}{2m(t)} + \frac{1}{2} m(t) w^2(t) x^2, \]

where \( m(t) \) and \( w(t) \) are the time-dependent mass and frequency, respectively. Our method is not based on the directly calculating of path integration but base on the using of a space-time transformation to simplify the path integration.

Materials and Methods

This section is the calculation of a harmonic oscillator with time-dependent mass and frequency described the Lagrangian [6]

\[ L(t) = \frac{1}{2} m(t) \dot{x}^2 - \frac{1}{2} m(t) w^2(t) x^2, \]  

where \( m(t) \) is the time-dependent mass and \( w(t) \) is the time-dependent frequency. By using the Euler-Lagrange equation for the Lagrangian in Eq.(1), the equation of motion can be written as

\[ \ddot{x} + 2 \frac{\dot{w}}{w} \dot{x} + w^2(t) x = 0, \]

where we define \( \rho(t) = \sqrt{m(t)} \).

By using the Pinney equation

\[ \ddot{a} + \frac{m(t)}{m(t)} \dot{a}(t) + w^2(t) a(t) = \frac{1}{m^2(t) \rho^2(t)} \]

the Lagrangian in Eq.(1) can be modified to

\[ L(t) = \frac{\rho m(t)}{a(t)} \dot{x}^2 + L_0, \]

where \( L_0 \) is

\[ L_0 = \frac{1}{2} \rho^2(t) \dot{x}^2 - \frac{1}{2} \dot{a}(t)^2 - \frac{1}{2} \rho^2(t) \dot{\rho}^2(t) \]

The next step is try to find a transformation that can transform the system with Lagrangian \( L_0 \) in Eq.(5) into the harmonic oscillator with unit mass and frequency. Let us consider the following transformation, which is the space and time transformation,

\[ y(t) = \frac{\dot{x}(t)}{a(t)}, \]

\[ dt = \frac{a(t)}{\rho(t)^2(t)} \]

By using space and time transformation, the Lagrangian \( L_0 \) in Eq. (5) can be written as

\[ \text{Atomic Physics} \]
The Feynman propagator \( K(x',t',x',t') \) is defined as the path integral [11],

\[
K(x',t',x',t') = \int \exp \left( \frac{i}{\hbar} \int_{t'}^{t} L \, dt \right) \, Dx(t),
\]

where \( Dx(t) \) is the path differential measure indicating that integrations are over all possible paths beginning at \( x(t') = x' \) and terminating at \( x(t') = x' \).

By substituting the Lagrangian in Eq.(4) into Eq.(9), the propagator reads as

\[
K(x',t',x',t') = K_0 \exp \left( \frac{i}{\hbar} \left[ m' \frac{a'}{a} x'^2 - \frac{m}{a} x^2 \right] \right),
\]

where \( K_0 \) is the new propagator corresponding to the new Lagrangian

\[
K_0 = \int \exp \left( \frac{i}{\hbar} \int_{t'}^{t} L_0 \, dt \right) \, Dx(t).
\]

If we now introduce a new time \( \tau \) in Eq. (7)

\[
\tau(t) = \int_{t'}^{t} \frac{1}{m(s)} \frac{ds}{a^2(s)},
\]

the action integral in Eq.(11) takes the form

\[
\int_{t'}^{t} L_0 \, dt = \int_{t'}^{t} L_0 \, d\tau,
\]

where \( L_0 \) is the unit mass and frequency oscillator Lagrangian in Eq. (8).

Using a process similar to Lawande and Dhara, we obtain the transformation of the action as follows

\[
Dx(t) = \frac{1}{\sqrt{\alpha a}} Dy(\tau).
\]

So, the propagator in Eq. (10) can be written as

\[
K(x',t',x',t') = \frac{1}{\sqrt{\alpha a}} \exp \left( \frac{i}{2\hbar} m' \frac{a'}{a} x'^2 - \frac{m}{a} x^2 \right) \left[ \frac{m'}{m} \frac{a}{a'} \right] K_0(y',\tau',y',\tau'),
\]

where \( K_0(y',\tau',y',\tau') \) is the propagator for harmonic oscillator with unit mass and frequency described by [11].

\[
K_0(y',\tau',y',\tau') = \int \exp \left( \frac{i}{\hbar} \int_{\tau'}^{\tau} L_0 \, d\tau \right) \, Dy(\tau)
= \left( \frac{1}{2 \sqrt{2 \sin \left( \tau - \tau' \right)}} \right)^{\frac{1}{2}} \exp \left( \frac{i}{2 \sqrt{2 \sin \left( \tau - \tau' \right)}} \left[ \frac{y'^2}{2} + y'^2 \cos \left( \tau - \tau' \right) - 2y'y' \right] \right).
\]

Substituting Eq. (16) into Eq. (15), the result is

\[
K(x',t',x',t') = \left( \frac{1}{2 \sqrt{2 \sin \left( \tau - \tau' \right)}} \right)^{\frac{1}{2}} \exp \left( \frac{i}{2 \sqrt{2 \sin \left( \tau - \tau' \right)}} \left[ \frac{y'^2}{2} + y'^2 \cos \left( \tau - \tau' \right) - 2y'y' \right] \right).
\]

The final step is rewriting Eq. (17) into the original variables as

\[
K(x',t',x',t') = \left( \frac{1}{2 \sqrt{2 \sin \left( \tau - \tau' \right)}} \right)^{\frac{1}{2}} \exp \left( \frac{i}{2 \sqrt{2 \sin \left( \tau - \tau' \right)}} \left[ \frac{y'^2}{2} + \frac{y'^2}{2} \cos \left( \tau - \tau' \right) - y'y' \right] \right).
\]

This result is the same form as the report of S. Pepore and B. Sukbot by using of Schwinger method [7].

**Results and Discussion**

In this article we have successfully calculated the Feynman propagator for a harmonic oscillator with time-dependent mass and frequency by the path integral method connecting with a space-time transformation. The resulting propagator in Eq.(18) is the same as in the report of Pepore et.al. [6-8]. The important step in this paper is to find the space-time Transformations in Eq.(6) and Eq.(7) and to write the Lagrangian interms of a unit mass and frequency oscillator in Eq.(6). The advantage of our method in this paper is that it can transform complicated system into a simplified problem. We have conclude here that our method is the effective method for solving the time-dependent problems because it requires some basic integration. Finally, it may be suggested that the methods in this paper can be applied to complicated problems, such as a time-dependent linear potential and a charged harmonic oscillator in a time-dependent electromagnetic field.

**References**