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ON THE DIOPHANTINE EQUATION OF FORM $A^z + B^y = C^z$

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Abstract—In this paper, we study some diophantine equation $A^x + B^y = C^z$ where $A$, $B$ and $C$ are prime and $x$, $y$ and $z$ are non-negative integers.

Keywords—diophantine equations; exponential equations; Catalan’s conjecture

I. INTRODUCTION

D. J. Sander ([4] and [5]) studied two diophantine equations $3^x + 3^y = 6^z$ and $4^x + 18^y = 22^z$. Then, Singh and Chotchaisthit [8] found two solutions of the diophantine equation $2^x + 5^y = z^2$. They noted that this equation has exactly two solutions with positive integer $(x, y, z) \in \{(3, 0, 3), (2, 1, 3)\}$. In the same paper, Suvarnamani, Singta and Chotchaisthit [8] found two solutions of the diophantine equation $4^x + 7^y = z^2$ for $x = 1$. After that Suvarnamani [7] found two solutions of the diophantine equations $2^x + 13^y = z^2$ and $2^x + 17^y = z^2$. Then Suvarnamani [6] studied the diophantine equation $4^x + p^y = z^2$ where $p$ is a prime number and $x$, $y$ and $z$ are non-negative integers.

In this research, we study the diophantine equation of form $A^x + B^y = C^z$ where $A$, $B$ and $C$ are prime and $x$, $y$ and $z$ are non-negative integers.

II. MAIN RESULTS

In this study, we use Catalan’s conjecture (see [2]). It is known that the only solution in integers $a > 1$, $b > 1$ and $y > 1$ of the equation $a^x - b^y = 1$ is $a = 3$ and $b = c = 2$. Now we have the following theorem.

Theorem 1. Consider the diophantine equation

$$p^x + p^y = q^z \tag{1}$$

where $p$ and $q$ are distinct prime numbers and $x$, $y$ and $z$ are non-negative integers. We get

(i) $(x, y, z) \in \{(0, 3, 2), (3, 0, 2)\}$ is a solution of the diophantine equation (1) for $p = 2$ and $q = 3$.

(ii) $(x, y, z) = (0, 0, 1)$ is a solution of the diophantine equation (1) for $q = 2$.

(iii) $(x, y, z) \in \{(0, k, 1)|k$ is a non-negative integer $\}$

$\cup \{(1, 0, k)|k$ is a non-negative integer $\}$ is a solution of the diophantine equation (1) for $p = q^k - 1$.

(iv) $(x, y, z) \in \{(0, 1, k)|k$ is a non-negative integer $\}$

$\cup \{(k, 0, 1)|k$ is a non-negative integer $\}$ is a solution of the diophantine equation (1) for $q = 1 + p^k$.

Proof: Consider the diophantine equation

$$p^x + p^y = q^z. \tag{2}$$

Suppose that $p$ and $q$ are distinct prime numbers.

Case 1: $x \leq y$. The diophantine equation (1) becomes $1 + p^{y-x} = q^2/p^x$. Thus $q^2/p^x$ must be an integer. Then $x = 0$. It follows that $q^2 - p^y = 1$.

By Catalan’s conjecture, we get $(x, y, z) = (0, 3, 2)$ is a solution of the diophantine equation (1) where $p = 2$ and $q = 3$.

If $z = 1$, then $q = 1 + p^k$. So, $(x, y, z) = (0, k, 1)$ is a solution of the diophantine equation (1) where $k$ is a non-negative integer such that $q = 1 + p^k$.

If $y = 0$, then $q^2 = 2$. So, $(x, y, z) = (0, 0, 1)$ is a solution of the diophantine equation (1) where $q = 2$.

If $y = 1$, then $p = q^k - 1$. So, $(x, y, z) = (1, 1, k)$ is a solution of the diophantine equation (1) where $k$ is a non-negative integer such that $p = q^k - 1$.

Case 2: $x > y$. The diophantine equation (1) becomes $p^{y-x} + 1 = q^2/p^x$. So $q^2/p^x$ must be an integer number. Then $y = 0$. It follows that $q^2 - p^x = 1$.

By Catalan’s conjecture, we get $(x, y, z) = (3, 0, 2)$ is a solution of the diophantine equation (1) where $p = 2$ and $q = 3$.

If $z = 1$, then $q = 1 + p^x$. So, $(x, y, z) = (k, 0, 1)$ is a solution of the diophantine equation (1) where $k$ is a non-negative integer such that $q = 1 + p^x$.
non-negative integer such that \( q = 1 + p^k \).

If \( x = 0 \), then \( q^2 = 2 \). So, \((x, y, z) = (0, 0, 1)\) is a solution of the diophantine equation (1) where \( q = 2 \).

If \( x = 1 \), then \( p = q^k - 1 \). So, \((x, y, z) = (1, 0, k)\) is a solution of the diophantine equation (1) where \( k \) is a non-negative integer such that \( p = q^k - 1 \).

**Theorem 2.** Consider the diophantine equation

\[ p^x + q^y = q^z \]  \( (2) \)

where \( p \) and \( q \) are distinct prime numbers and \( x, y \) and \( z \) are non-negative integers. We get

(i) \((x, y, z) = (3, 0, 2)\) is a solution of the diophantine equation (2) for \( p = 2 \) and \( q = 3 \).

(ii) \((x, y, z) = (0, 0, 1)\) is a solution of the diophantine equation (2) for \( q = 2 \).

(iii) \((x, y, z) \in \{(1, 0, k) | k \text{ is a non-negative integer}\}\) is a solution of the diophantine equation (2) for \( p = q^k - 1 \).

(iv) \((x, y, z) \in \{(k, 0, 1) | k \text{ is a non-negative integer}\}\) is a solution of the diophantine equation (2) for \( q = 1 + p^k \).

**Proof:** Consider the diophantine equation

\[ p^x + q^y = q^z \]

where \( p \) and \( q \) are distinct prime numbers and \( x, y \) and \( z \) are non-negative integers.

Case 1: \( y \geq z \). We get \( q^y \geq q^z \) and \( p^x > 0 \). So, \( p^x + q^y > q^z \). That is the diophantine equation (2) has no solution.

Case 2: \( y < z \). Since \( q^z \equiv 0 \pmod{q} \) and \( p^x + q^y \equiv 0 \pmod{q} \) except when \( y = 0 \), so \( p^x + q^y = q^z \) is impossible except when \( y = 0 \).

If \( y = 0 \), the diophantine equation (2) becomes \( q^z = p^x \).

By Catalan's conjecture, we get \((x, y, z) = (3, 0, 2)\) is a solution of the diophantine equation (2) where \( p = 2 \) and \( q = 3 \).

If \( x = 0 \), then \( q^2 = 2 \). So, \((x, y, z) = (0, 0, 1)\) is a solution of the diophantine equation (2) where \( p = 2 \).

If \( x = 1 \), then \( p = q^k - 1 \). So, \((x, y, z) = (1, 0, k)\) is a solution of the diophantine equation (2) where \( k \) is a non-negative integer such that \( p = q^k - 1 \).

**Theorem 3.** If \( p, q \) and \( r \) are distinct primes which are not 2, then the diophantine equation

\[ p^x + q^y = r^z \]

has no solution.

**Proof:** Consider the diophantine equation \( p^x + q^y = r^z \). Since \( p, q \) and \( r \) are odd, so \( p^x, q^y \) and \( r^z \) are too. Then, \( p^x + q^y \) is even. So, \( p^x + q^y \) can not be a prime. Hence the diophantine equation (3) has no solution.

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